Quantum Physics I. Nov. 1, 2018.

1] a) $\phi(k) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{+\infty} \psi(x) \cdot e^{-ikx} \cdot dx$ 5)

b) $\phi(k) = \frac{1}{\sqrt{2\pi}} \left(\frac{2a}{\pi}\right)^{1/4} \cdot \int e^{-a\left(x+\frac{ik}{2a}\right)^2} \cdot e^{-k^2/4a} \cdot dx$

complete square 5)

$= \underline{\qquad} \cdot 2 \cdot \sqrt{\pi} \cdot \left(\frac{1}{2\sqrt{a}}\right) \cdot e^{-k^2/4a}$

$= \left(\frac{1}{2a\pi}\right)^{1/4} \cdot e^{-k^2/4a}$ ← Gaussian 5)

c) $\sigma_x^2 = \langle x^2 \rangle = 2\sqrt{\pi} \cdot 2 \cdot \left(\frac{1}{2\sqrt{a}}\right)^3 \cdot \left(\frac{2a}{\pi}\right)^{1/2}$ 5)

$= \frac{1}{\sqrt{2}} \cdot \frac{1}{a}$

$\sigma_p^2 = \hbar^2 \cdot \langle k^2 \rangle = \frac{1}{\sqrt{2a\pi}} \cdot 2 \cdot \sqrt{\pi} \cdot 2 \cdot \left(\frac{2\sqrt{a}}{2}\right)^3 \cdot \hbar^2$

$= \frac{\cancel{2\sqrt{a}} \cdot a \cdot \hbar^2}{2\sqrt{2}}$ (saturates Heisenberg). 5)

d) free particle ⇒ momentum conserved

⇒ σ_p stays the same.

different momenta ⇒ spatial distrib. changes

⇒ σ_x increases. 5)

2] a) $m \cdot \frac{d}{dt}\langle x \rangle$

$$= m \int \left(\frac{\partial \psi^*}{\partial t} \cdot x \psi + \psi^* \cdot x \cdot \frac{\partial \psi}{\partial t} \right) dx$$

$$= m \cdot -\frac{i}{\hbar} \cdot \int \left(-H\psi^* \cdot x\psi + \psi^* x \, H\psi \right) dx \quad \text{s)}$$

($H = K + V$, V drops out)

($K = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$, integr by parts)

$$= \frac{m}{2m} \cdot -i\hbar \cdot 2 \int \left(\psi^* \cdot \frac{\partial}{\partial x} \psi \right) dx \quad \text{5)}.$$

$$= -i\hbar \int \psi^* \frac{\partial}{\partial x} \psi \cdot dx = \langle p \rangle$$

b) $\frac{d}{dt}\langle p \rangle$

$$= -i\hbar \int \left(\left(\frac{\partial \psi^*}{\partial t} \right) \cdot \frac{\partial \psi}{\partial x} + \psi^* \cdot \frac{\partial}{\partial x} \frac{\partial \psi}{\partial t} \right) dx \quad \text{4)}$$

$$= - \int \left(-H\psi^* \frac{\partial \psi}{\partial x} + \psi^* \frac{\partial}{\partial x} H\psi \right) dx$$

($H = K + V$, K drops out) 4)

$$= - \int \psi^* \cdot \frac{\partial V}{\partial x} \cdot \psi \, dx = - \langle V' \rangle.$$

$\rightsquigarrow$ Newton's law. 2).

c) ∞-dim VS.

<u>discrete</u>. eigenvalues real ✓
eigenfunctions normalisable ✓

e.g. Ham. of infinite square well. 5)

<u>continuous</u> eigenvalues real: not guaranteed, assumption,
normalisable: NO. (Dirac normalisable) 5)

e.g. Ham. of free particle

3] a) eigenfunctions $\sqrt{\frac{1}{2\pi}}\, e^{+im\phi}$ periodic in 2π

values m. $\phi \to \phi + 2\pi$. 5)

b) ground state: $\hat{H} = \sqrt{\frac{1}{\pi}}$. eigenvalue 0. 5)

c) First excited state:

$\hat{H} = \cos(\theta)$ eigenvalue $-2 = -l(l+1)$.

periodic in π. 5)

4] a) pion = scalar particle
anti-symm. singlet } $S = 0$. 5)

b) $|\uparrow\rangle_z = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|\uparrow\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$|\downarrow\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ 5)

$|\uparrow\rangle_z = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_x + |\downarrow\rangle_x \right)$.

c). probabilities:

if electron is $|\uparrow\rangle_z \Rightarrow$ 50/50 for x-spin of positron.

$|\downarrow\rangle_z \Rightarrow$ same.

(no correlation). 5)